Optimal choice of health and retirement in a life-cycle model

Michael Kuhn, VID and Wittgenstein Centre

Stefan Wrzaczek, TU Vienna, VID and Wittgenstein Centre

Alexia Prskawetz, TU Vienna, VID and Wittgenstein Centre

Gustav Feichtinger, TU Vienna , VID and Wittgenstein Centre

September 4th 2012, Seminar on Economic Demography, Paris

1. Introduction

- Empirical studies on health and retirement but only few theoretical works on the relationship between health and retirement

- simultaneous decision on health and retirement

$$
\begin{tikzcd}
\text{health} \arrow[d, shift right=2] \\
\text{retirement} \arrow[u, shift right=2]
\end{tikzcd}
$$

longevity
&
morbidity/productivity

(life-cycle) income
&
returns to health

- Literature: Bloom et al. 2007 “A Theory of Retirement”
  d’Albis et al. 2012 “ Mortality transition and differential incentives for early retirement”
  Galama et al. 2008 “Grossman’s Health Threshold and Retirement”

However:

Bloom et al. 2007, d’Albis et al. 2012:

exogenous variation of health on retirement

Galama et al. 2008:

endogenous health but do not connect it to survival (only morbidity/productivity effect)

In this paper:

longevity – health - retirement nexus

with **endogenous** health (morbidity and mortality) +
endogenous retirement decisions

Investigate

the relationship between (optimal) health and retirement when health relates both to

mortality/survival => pull for longer working life
morbidity/disutility of labour => push for longer working life

the implications of health-related moral hazard when annuity returns do not adjust to individual health (Davies & Kuhn 1992, Philipson & Becker 1998)

2. The Model

Objective: maximize lifetime utility

2 phases of life: working life + retirement (at age τ)

utility in first phase (t ≤ τ): $u(c(t)) - v(t, S(t))$

utility in second phase (t > τ): $u(c(t))$

benefit from consumption c: $u(c(t)) \quad u' > 0, u'' < 0, u'(0) = +\infty$

disutility from work: $v(t, S(t)) \quad v_t \geq 0, v_{tt} \geq 0$

Disutility from work is responsive to health S(t): $v_S \leq 0, v_{SS} \geq 0$

with the boundary case: $v_S = 0$

Health:

Stock of health = survival through age t $S(t)$ with $\dot{S}(t) = -\mu(t, h(t))S(t), S(t_0) = 1$

Mortality = rate of depreciation ↓ in health care h $\mu(t, h(t))$ $\mu_h < 0, \mu_{hh} > 0,$

$\mu_h(t,0) = -\infty$

Moral hazard within the annuity market:

Annuity return: $r + \theta\bar{\mu}(t) + (1-\theta)\mu(t, h(t))$ with $\theta \in [0,1]$

with r = market interest

$\theta = 0$ → Perfect annuity market: individualised return

$\theta = 1$ → Moral hazard: individual takes return as given

In equilibrium: $\bar{\mu}(t) = \mu(t, h(t))$

The Full Model

$$\max_{c(t),h(t),\tau} \int_{t_0}^{\tau} e^{-\rho t} S(t)(u(c(t)) - \nu(t, S(t)))dt + \int_{\tau}^{T} e^{-\rho t} S(t)u(c(t))dt$$

subject to

$$\begin{aligned}
\dot{A}(t) &= w(t) - c(t) - h(t) + (r + \theta\bar{\mu} + (1-\theta)\mu)A(t), \quad A(t_0) = 0 \quad \text{for } t \leq \tau \\
\dot{A}(t) &= -c(t) - h(t) + (r + \theta\bar{\mu} + (1-\theta)\mu)A(t), \quad A(T) = 0 \quad \text{for } t \geq \tau \\
\dot{S}(t) &= -\mu(t, h(t))S(t), \quad S(t_0) = 1
\end{aligned}$$

With ρ = rate of time preference

Two state variables:	Assets	A(t)
	Health	S(t)
Three controls:	Consumption	c(t)
	Health care	h(t)
	Retirement	τ

First-best allocation: $\theta = 0$

3. Optimal allocation

Consumption: $$\frac{u_c(c^*(t))}{u_c(c^*(s))e^{\rho(s-t)}} = e^{r(s-t)}$$

Euler: MRIS = compound interest

Health: $$-\frac{1}{\mu_h(h^*(t))} = \psi^i(t)$$

Cost of increasing S(t) by one unit = Value of health/survival

Retirement: $$\frac{\nu(\tau^*, S(\tau^*))}{u_c(c^*(\tau^*))} = w(\tau^*)$$

Value of disutility = earnings

From Euler: $$c^*(t) = c_0 e^{(r-\rho)(t-t_0)}$$

Value of health (VOH) = WTP for an increase in S(t) at age t

Working life :

$$\psi^1(t) \; : \; = \psi(t \leq \tau) = \int_t^{\tau} e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{u(c(s)) - v(s, S(s))}{u_c(c(s))} ds$$
$$+ \int_{\tau}^{T} e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{u(c(s))}{u_c(c(s))} ds$$

Gross surplus of survival

Value of health (VOH) = WTP for a small reduction in μ at age t

Working life :

$$\psi^1(t) \; : = \psi(t \leq \tau) = \int_t^{\tau} e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{u(c(s)) - v(s, S(s))}{u_c(c(s))} ds$$

$$+ \int_{\tau}^{T} e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{u(c(s))}{u_c(c(s))} ds$$

$$- \int_t^{T} e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{S(s)\nu_S}{u_c(c(s))} ds$$

Gross surplus of survival

Value of morbidity reduction

Value of health (VOH) = WTP for a small reduction in μ at age t

Working life :

Gross surplus of survival

Value of morbidity reduction

$$\psi^1(t) \; : \; = \psi(t \leq \tau) = \int_t^\tau e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{u(c(s)) - v(s, S(s))}{u_c(c(s))} ds$$
$$+ \int_\tau^T e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{u(c(s))}{u_c(c(s))} ds + (1-\theta)\left[H(t) - E(t)\right]$$
$$- \int_t^T e^{-r(s-t)} \frac{S(s)}{S(t)} \frac{S(s)\nu_S}{u_c(c(s))} ds$$

with

Human wealth:

$$H(t) \; : \; = \int_t^T e^{-r(s-t)} \frac{S(s)}{S(t)} w(s)\, ds$$

Future expenditure

$$E(t) \; : \; = \int_t^T e^{-r(s-t)} \frac{S(s)}{S(t)} \left[c(s) + h(s)\right] ds$$

Value of health (VOH) = WTP for a small reduction in μ at age t

Working life :

$$\psi^1(t) \;:=\psi(t\leq\tau)=\int_t^T e^{-r(s-t)}\frac{S(s)}{S(t)}\frac{u(c(s))-v(s,S(s))}{u_c(c(s))}ds$$

Gross surplus of survival

$$+\int_\tau^T e^{-r(s-t)}\frac{S(s)}{S(t)}\frac{u(c(s))}{u_c(c(s))}ds+(1-\theta)\left[H(t)-E(t)\right]$$

Value of morbidity reduction

$$-\int_t^\tau e^{-r(s-t)}\frac{S(s)}{S(t)}\frac{S(s)\nu_S}{u_c(c(s))}ds$$

with

Human wealth:

$$H(t)\;:=\int_t^T e^{-r(s-t)}\frac{S(s)}{S(t)}w(s)\,ds$$

Future expenditure

$$E(t)\;:=\int_t^T e^{-r(s-t)}\frac{S(s)}{S(t)}\left[c(s)+h(s)\right]ds$$

Retirement:

$$\psi^2(t)\;:=\psi(t\geq\tau)=\int_t^T e^{-r(s-t)}\frac{S(s)}{S(t)}\frac{u(c(s))}{u_c(c(s))}ds-(1-\theta)E(t)$$

Life cycle complementarity between

HEALTH and CONSUMPTION

HEALTH and RETIREMENT

RETIREMENT and CONSUMPTION

Properties of health care :

→ Complementarity with (future) health if and only if θ=1:

$$\frac{\partial h(t)}{\partial h(\hat{t})}\Big|\hat{t} \in (t,T] = \theta e^{-r(\hat{t}-t)}\frac{S(\hat{t})}{S(t)} \geq 0$$

→ Complementarity with (past) health if morbidity matters

$$\frac{\partial h(t<\tau)}{\partial h(\hat{t})}\Big|\hat{t} \in [t_0,t) = \mu(h(\hat{t}))\int_t^{\tau}\frac{v_S + Sv_{SS}}{u_c}e^{-r(s-t)}\frac{S(s)}{S(t)}ds \geq 0$$

→ Complementarity with consumption

$$\frac{\partial h(t \geq \tau)}{\partial c_0} = e^{(r-\rho)(t-t_0)}\int_t^T e^{-\rho(s-t)}\frac{S(s)}{S(t)}\left(\frac{-uu_{cc}}{u_c}+\theta\right)ds > 0$$

$$\frac{\partial h(t<\tau)}{\partial c_0} = e^{(r-\rho)(t-t_0)}\int_t^{\tau} e^{-\rho(s-t)}\frac{S(s)}{S(t)}\left(\frac{-(u-v)u_{cc}}{u_c}+\theta\right)ds + e^{-r(\tau-t)}\frac{S(\tau)}{S(t)}\frac{\partial \psi^2(t)}{\partial c_0} > 0$$

→ Complementarity with retirement:

$$\frac{\partial h(t \geq \tau)}{\partial \tau} = 0$$

$$\frac{\partial h(t < \tau)}{\partial \tau} = -e^{-r\left(\tau^{*}-t_0\right)}\frac{S\left(\tau^{*}\right)}{S(t)}\left(\overbrace{\frac{v_S S\left(\tau^{*}\right)}{u_c}}^{<0} + \theta \overbrace{\frac{v\left(\tau^{*}, S\left(\tau^{*}\right)\right)}{u_c}}^{\geq 0}\right) \geq 0 \Leftrightarrow \eta(v,S) \geq \theta$$

where $\eta(v,S) := -v_S\, S/v$

Result 1:

(i) First-best θ=0: Pre-retirement health is complementary with retirement age.

(ii) Second-best θ=1: Pre-retirement health is complementary with retirement age if and only if η(v,S)>1.

4 Moral Hazard in the Annuity Market

Assumption: A(t)>0 holds for all t.

Then,…

$$\frac{\partial h(t)}{\partial \theta} = -[H(t) - E(t)] = A(t) > 0 \Leftrightarrow A(t) > 0$$

Result 2: Exogenous retirement (Kuhn & Davies 1992, Philipson & Becker 1998)

For a given level of retirement, moral hazard in the annuity market ↑ health expenditure and ↓ consumption for all t.

Result 3: Endogenous retirement

For an endogenous level of retirement, moral hazard in the annuity market ↑ health expenditure for all t, ↑ the retirement age and

(i) ↓ consumption for all t if the disutility of labour is relatively unresponsive to health but…

(ii) ↑ consumption for all t if the disutility of labour is relatively responsive to health

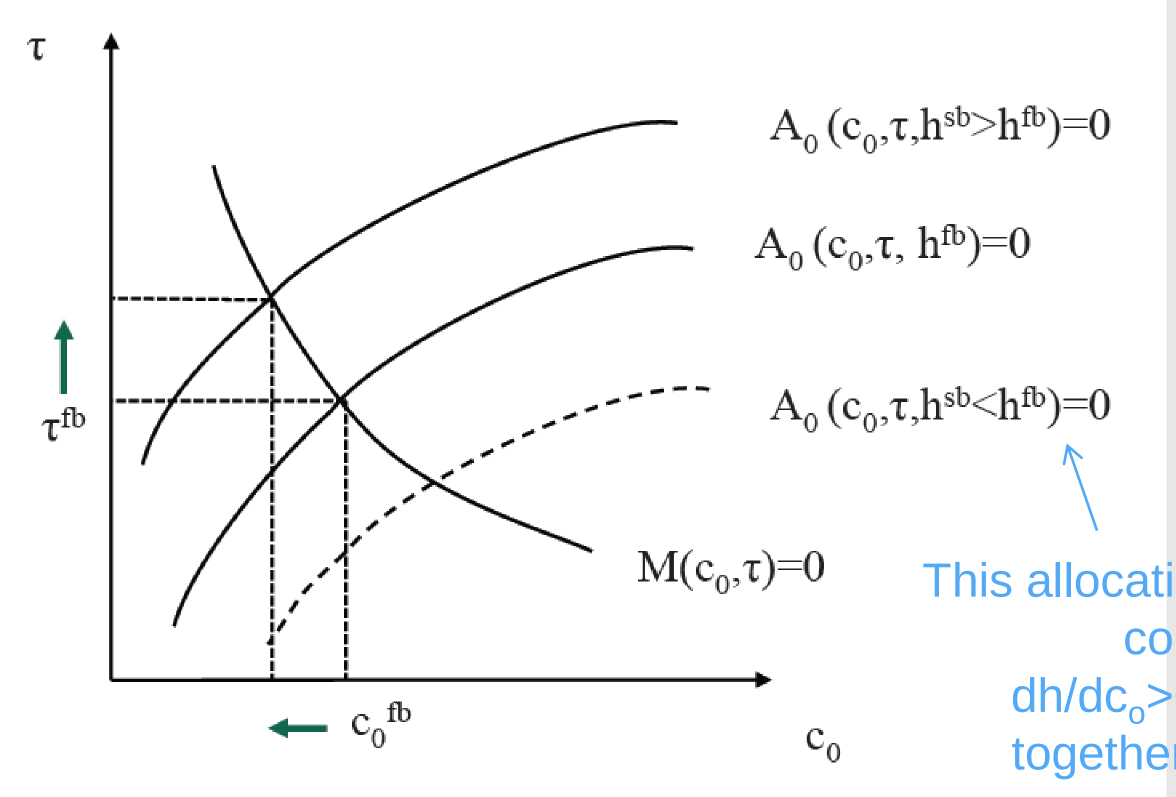
optimality locus

$$M\left(c_0, \tau\right) := w\left(\tau\right) - \frac{\nu(\tau)}{u_c(c_0 e^{(r-\rho)(\tau - t_0)})} = 0$$

feasibility locus

$$A_0\left(c_0, \tau, h\right) := \int_{t_0}^{\tau^*} \Phi(s, t_0) w\left(s\right) ds - \int_{t_0}^{T} \Phi(s, t_0)\left(c\left(s\right) + h\left(s\right)\right) ds = 0$$

Case (i): v_S=0

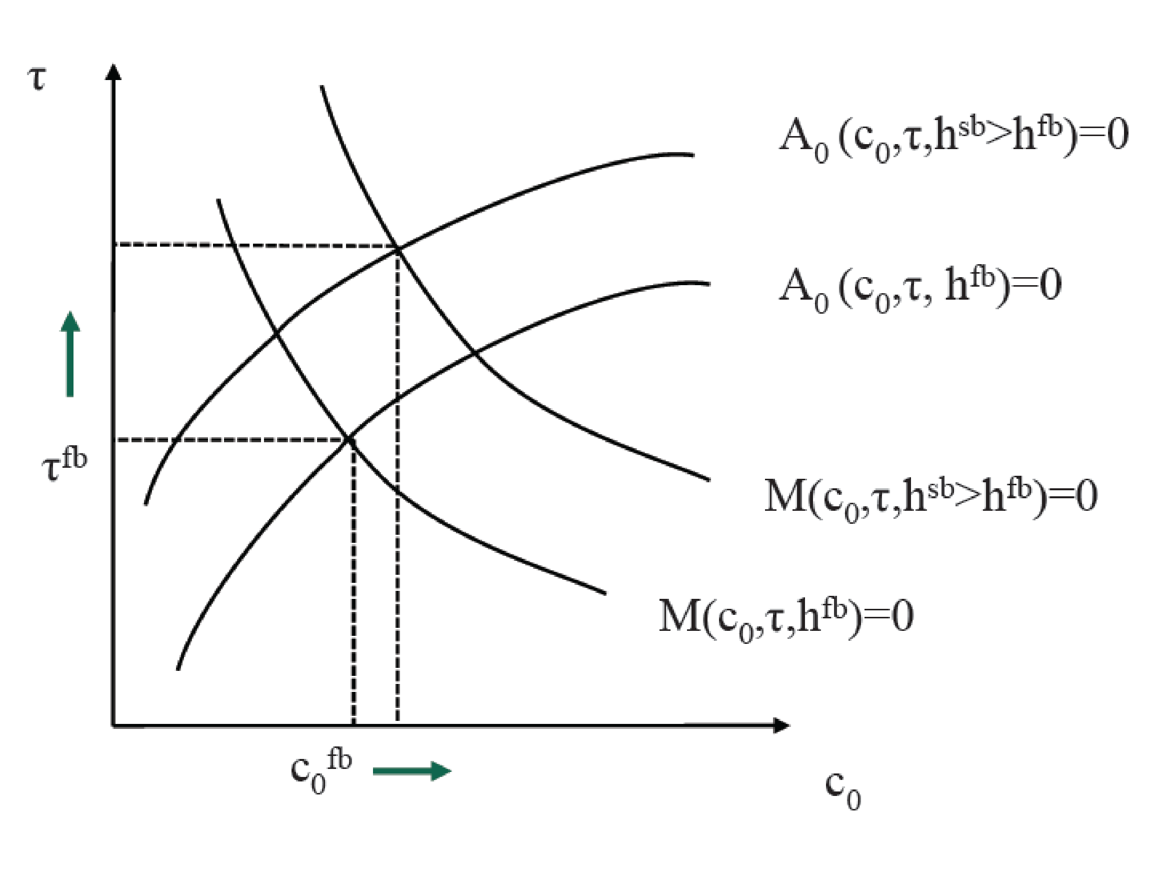


This allocation contradicts complementarity $dh/dc_o>0$ and $dh/d\tau<0$ together with $dh/d\theta>0$

Excessive health care => under-consumption

Mitigated (but not overturned) by an increase in retirement age.

Case (ii): $v_S<0$



Morbidity reduction => magnifies expansion of working-life

=> If strong enough => generate scope for extra consumption!

5. Numerical Results

$$u(c(t)) = b + \frac{c(t)^{1-\sigma}}{1-\sigma}$$

b=6; σ=1.5

from HMD (1990-2000)

$$\mu(t, h(t)) = \tilde{\mu}(t)\phi(t, h(t))$$

$$\phi(t, h(t)) = 1 - \sqrt{\frac{h(t)}{z}\frac{t-T}{1-T}}$$

z=30; T=110

w(t)= 52,630 (US average earnings 2000); r=ρ=0.06; α=0.2

Case (i): $v(t)=v(S(h^{fb}(t)))$ from case (ii) => identical (ex-post) disutility

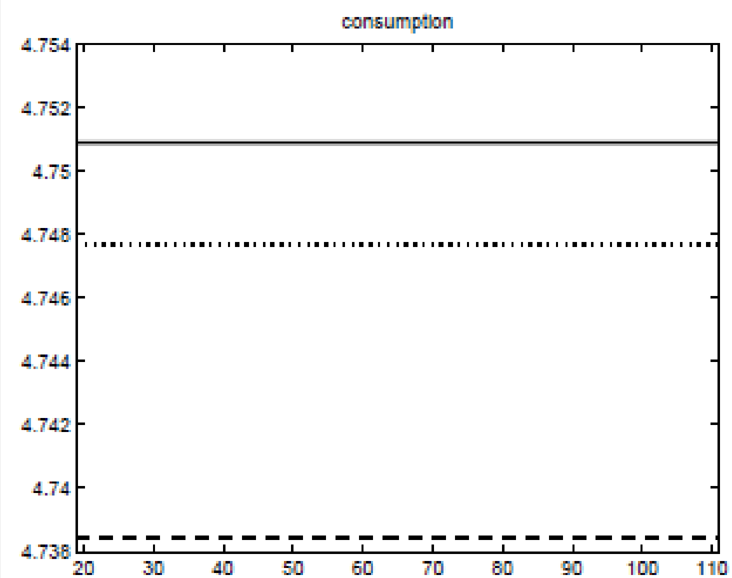
Case (ii): $$v(S) = \bar{z}(1-S)$$ z-bar=6.5

6 scenarios

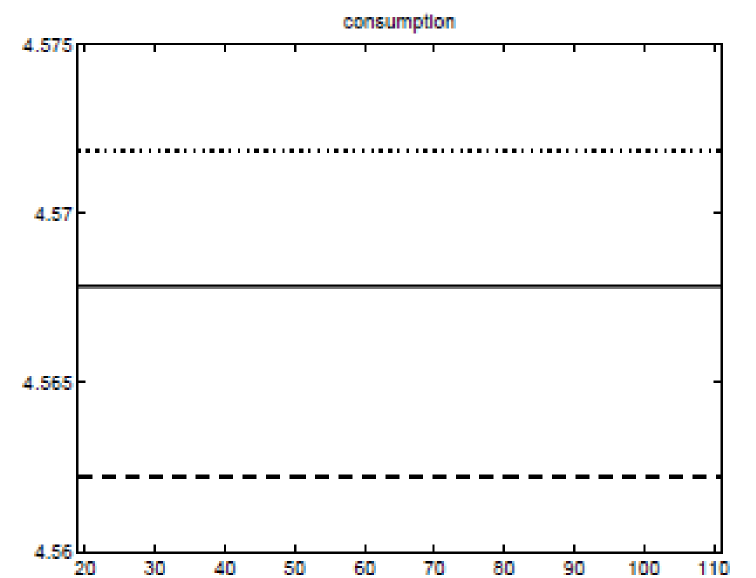
	$\nu(t,S) = \nu(t)$	$\nu(t,S) = \nu(S)$
first best ($\theta = 0$, $\tau = \tau^{fb}$)	1.1	2.1
moral hazard, exog. retirement ($\theta = 1$, $\tau = \tau^{fb}$)	1.2	2.2
moral hazard, edog. retirement ($\theta = 1$, $\tau = \tau^{sb}$)	1.3	2.3

Consumption:

Case (i): v(t)



Case (ii): v(S)



First-best: solid (scenario 1.1 and 2.1)

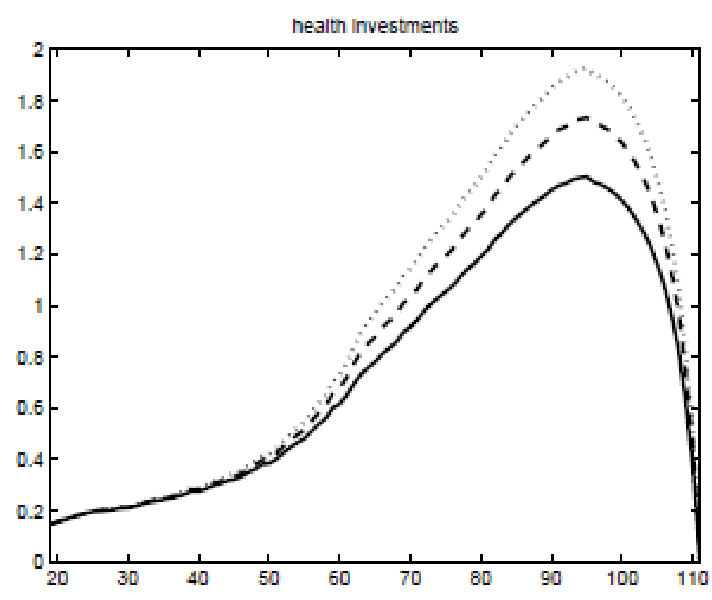
Moral hazard…with fixed τ: dashed (scenario 1.2 and 2.2);

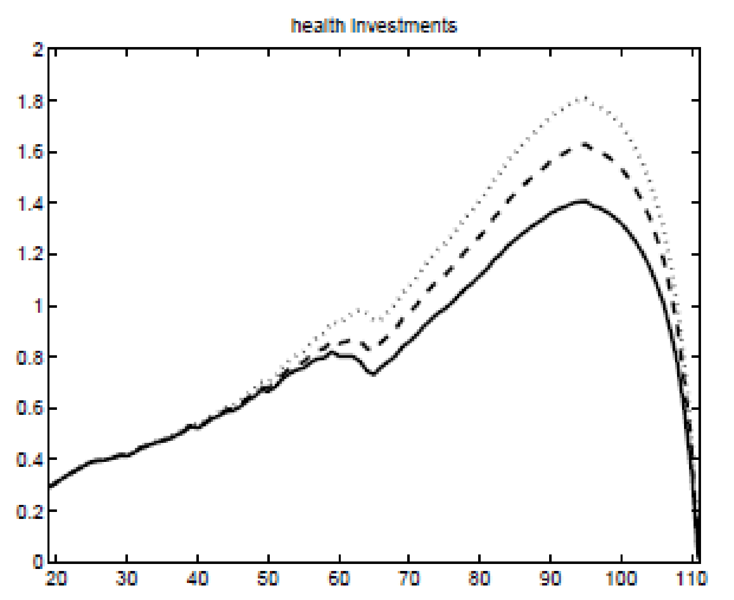
…with endogenous τ: dotted (scenario 1.3 and 2.3)

Health:

Case (i): v(t)

Case (ii): v(S)





First-best: solid (scenario 1.1 and 2.1)
Moral hazard…with fixed τ: dashed (scenario 1.2 and 2.2);
…with endogenous τ: dotted (scenario 1.3 and 2.3)

7. Conclusions

- Life-cycle framework to study in a unified way retirement & health (both with a mortality and morbidity dimension)

- Moral hazard on the annuity market => excessive health care.

- Weak morbidity => excessive working life and under-consumption

- Strong morbidity => excessive working life and over-consumption (due to ‘productivity effect’). Moral hazard is ‘magnified’.

- Mandatory / Early retirement as a second-best policy aimed at curtailing moral hazard

Thank You